**MATHEMATICS-IV**

**UNIT-**I :: Chapter-I ::**Determination of Roots of Non-linear Equations**

**Bisection Method:**(OR) **Bolzanowistrassmerhod** :

This method is based on the “intermediate value theorem” which states that “Suppose f(x) be a continuous function defined on [ a , b ] such that f(a) and f(b) have opposite signs then there exists at least one point c in ( a , b ) such that f(c) = 0”.

The method is applicable for solving the equation f(x) = 0 for the real variable x, where f is a continuous function defined on an interval [a, b] and f(a) and f(b) have opposite signs. In this case a and b are said to bracket a root since, by the intermediate value theorem,the continuous function f must have at least one root in the interval (a, b). At each step the method divides the interval in two by computing the midpoint c = (a+b) / 2 of the interval. If f(c) = 0 then c itself a root of the equation f(x) = 0 , otherwise the root lie between ‘a and c’ or ‘c and b’ depending on either f(a) and f(c) have opposite signs and bracket a root, or f(c) and f(b) have opposite signs and bracket a root.The method selects the subinterval that is a bracket as a new interval to be used in the next step. In this way the interval that contain s a zero of f is reduced in width by 50% at each step. The process is continued until the interval is sufficiently small.

**Problems:**Find a real root of the following equations up to 3 decimal places by using Bisection method.

(1) (1.5213623)(2) x3 + 3x – 5 (1.154296875)

(3) Find negative root of  (-2.7064) (4) ( 1.11328)

(5)  (0.5156) (6) ( 2.687)

(7) (0.686)(8)  (1,4036)

(9) (0.567)(10)  (1.3437)

**Regulafalsi method (or) False position method:**

The convergce process in the bisection method is very slow. It depends only on the choice of end points of the interval [a,b]. The function f(x) does not have any role in finding the point c (which is just the mid-point of a and b). It is used only to decide the next smaller interval [a,c] or [c,b]. A better approximation to c can be obtained byRegulafalsimethod,taking the straight line L joining the points (a,f(a)) and (b,f(b)) intersecting the x-axis.

(derive this)



**Problems:** Find a real root of the following equations up to 3 decimal places by using Regula- falsi method.

(1)  (2.0943) (2)  (0.5156)

(3) (3.789) (4)  (0.853)

(5) (0.6071) (6)  (0.3604)

(7) (2.798) (8)  (0.357)

**Newton-Raphson Method:**

 (Prove it )

Note: Newtos’s formula will converges if <in the interval considered.

**Problems:**

1. Find the positive root of correct to 3 decimal places, using N-R method.
2. Find the Newton’s Method , the real root of the equation 3x = cos x +1.
3. Using Newton’s iterative method, find the real root of  correct to 5 decimal places.
4. Evalute the following correct to 4 decimal places by N-R method.
5.  (ii)  (iii)  (iv)  (v) 
6. Find by Newton’s Method, a root of the following equations correct to 4 decimal places.
7.  (ii)  (iii)  (iv) 
8. (i)  (ii)  (iii)  (iv)  (4.5)

(v)  (vi) 

**ITERATION METHOD:**To find the root of the equation f(x)=0 by successive approximations , we rewrite the equation as  . The root of f(x) = 0 is same as the points of intersection of the straight line y = x and the curve .

 Let  be the initial approximation of the desired root of f(x) = 0 ,then the first approximation is given by . Now treating as the initial value, the second approximation is given by . Proceeding this way the n th approximation is given by .

Note: We choose the initial approximation  by satisfying the condition < 1

**Problems:**Using Iterative method, find the real root of correct to 3 decimal places.

1.  (2)  (3)  (4) Cos x = 3x – 1
2.  (6)  (7) 

**Chapter-II :: Curve Fitting**

**Method of Least Squares:**Let be the set of observed values and  be the set of true values of the function y = f (x) for i = 1, 2, 3,……. Let  be the error of the function corresponding to these points. The principle of least squares states that the sum of the squares of the errors is minimum.

i.e.  is minimum.

I) **Fitting a straight line y = ax + b:** Fit a straight line to the following data.

1) y = 13.6x

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| X | 1 | 2 | 3 | 4 | 5 |
| Y | 14 | 27 | 40 | 55 | 68 |

 2) Fit a straight line to the following data, and find y value corresponding to x=6. (A) y = 0.69x+11.27

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| X | 0 | 5 | 10 | 15 | 20 | 25 |
| Y | 12 | 15 | 17 | 22 | 24 | 30 |

3) y = 1.6x +2.1

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| X | 0 | 1 | 2 | 3 | 4 |
| Y | 2.1 | 3.5 | 5.4 | 7.3 | 8.2 |

4) Also estimate the production in 1976 (A) y= 0.06x+126.66, 11.1

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Years | 1951 | 1961 | 1971 | 1981 | 1991 |
| Production | 10 | 12 | 8 | 10 | 14 |

5) Also estimate the value of y at x = 70 (A) y= 0.4242x+37.90 ; 67.603

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| X | 71 | 68 | 73 | 69 | 67 | 65 | 66 | 67 |
| Y | 69 | 72 | 70 | 70 | 68 | 67 | 68 | 64 |

**II) Fitting a second degree polynomial (or) parabola :** Fit a parabola to the following data.

1. Fit a second degree polynomial by taking x as indeprndent variable.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| X | 0 | 1 | 2 | 3 | 4 |
| Y | 1 | 5 | 10 | 22 | 38 |

(A) 

(2)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| X | 10 | 12 | 15 | 23 | 20 |
| y | 14 | 17 | 23 | 25 | 21 |

1.  where X = x – 15; Y = y – 25

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| X | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
| Y | 1.1 | 1.3 | 1.6 | 2.0 | 2.7 | 3.4 | 4.1 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| X | 0 | 1 | 2 | 3 | 4 |
| Y | 1 | 1.8 | 1.3 | 2.5 | 2.3 |

1. 

(5)

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| Y | 14 | 18 | 23 | 29 | 36 | 40 |  |

1. 

( III**) Fitting a power curve of the form :** Fit a power curve of the form to the following data:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| X | 1 | 2 | 3 | 4 |
|  Y | 4 | 11 | 35 | 100 |

(a: 

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Year | 1951 | 1952 | 1953 | 1954 | 1955 | 1956 | 1957 |
| Production in tons | 201 | 263 | 314 | 395 | 427 | 504 | 612 |

(A:  )

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| X | 2 | 3 | 4 | 5 | 6 |
| Y | 8.3 | 15.4 | 33.1 | 65.2 | 127.4 |

(A:  )

(IV) **Fitting a power curve of the form  :**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| X | 61 | 26 | 7 | 2.6 |
| Y | 350 | 400 | 500 | 600 |

(A:  )

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| X | 1 | 2 | 3 | 4 |
| Y | 1.65 | 2.70 | 4.50 | 7.35 |

(A:  )

The pressure and volume of a gas are related by the equation . Fit this equation for the following data using the principle of least squares.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| P | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 |
| V | 1.62 | 1.00 | 0.75 | 0.62 | 0.52 | 0.46 |

(A:  )

**Correlation:**The changes in one variable are associated or followed by changes in the other variable is called correlation .Such data connecting two variable is called bivariate population.

Ex: The yield of a crop varies with the amount of rainfal ,

 If an increase (or decrease) in the values of one variable corresponds to an increase (or decrease) in the other, the correlation is said to be positive. If an increase (or decrease) in the values of one variable corresponds to an decrease (or increase) in other, the correlation is said to be negative. If there is no relationship indicated between the variables , they are said to be independent or uncorrelated.

Coefficient of Correlation: The numerical measure of correlation is called the coefficient of correlation. It is denoted by (or) r and defined as



the coefficient of correlation is always lie between-1 to +1 . If -1 < r < 0 then the correlation is negative, if 0< r < 1 then the correlation is positive and if r = 0 then there was no correlation.

**Problems:**

1. Psychological tests of intelligence and of engineering ability were applied to 10 students . Here is a record of ungrouped data showing intelligence ratio (I.R.) and engineering ratio (E.R.) . Calculate the coefficient of correlation. ( 0.59 )

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| I.R. | 105 | 104 | 102 | 101 | 100 | 99 | 98 | 96 | 93 | 92 |
| E.R. | 101 | 103 | 100 | 98 | 95 | 96 | 104 | 92 | 97 | 94 |

1. Find the correlation coefficient between a and y from the given data: ( 0.96 )

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| X | 78 | 89 | 97 | 69 | 59 | 79 | 68 | 57 |
| Y | 125 | 137 | 156 | 112 | 107 | 138 | 123 | 108 |

**Another method to find correlation coefficient:**



Where ,  ,

**Problems:**

(1) Calculate the correlation coefficient for the following heights (in inches) of fathers(X) and their sons (Y). ( 0.603 )

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| X | 65 | 66 | 67 | 67 | 68 | 69 | 70 | 72 |
| Y | 67 | 68 | 65 | 68 | 72 | 72 | 69 | 71 |

(2) The heights and weights of 5 students are given below . Find the correlation coefficient. ( 0.98 )

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Heights (c.m.) | 160 | 161 | 162 | 163 | 164 |
| Weights (k.g.) | 50 | 53 | 54 | 56 | 57 |

**Rank correlation Coefficient:**A group of n individuals may be arranged in order to merit with respect to some characteristic. The same group give different orders for different characteristics. Considering the orders corresponding to two characteristics A and B , the correlation between these n pairs of ranks is called “ rank correlation “ in the characteristics A and B for that group of individuals.

; Where di = rank xi – rank yi

**Problems:**

(1) Find the rank correlation for the following data: ( 0.932 )

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| X | 56 | 42 | 72 | 36 | 63 | 47 | 55 | 49 | 38 | 42 | 68 | 69 |
| Y | 147 | 125 | 160 | 118 | 149 | 128 | 150 | 145 | 115 | 140 | 152 | 155 |

(2) ( 0.545 )

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| X | 68 | 64 | 75 | 50 | 64 | 80 | 75 | 40 | 55 | 64 |
| Y | 62 | 58 | 68 | 45 | 81 | 60 | 68 | 48 | 50 | 70 |

(3) The participants in a contest are ranked by two judges as follows:

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| X | 1 | 6 | 5 | 10 | 3 | 2 | 4 | 9 | 7 | 8 |
| Y | 6 | 4 | 9 | 8 | 1 | 2 | 3 | 10 | 5 | 7 |

Calculate the rank correlation coefficient .( 0.6 )

**Regression of Lines:**

(1) The equation of Regression line of x on y isWhere is called regression coefficient of x on y. It is defined as . Here  and  are standard deviations of x and y respectively.

(2) The equation of Regression line of y on x 

(3) The Sign of ,and  are always same .

(4) The two regression lines x on y and y on x always passes through the point .

(5) 

**Problems:**

(1) Calculate the coefficient of correlation from the following data . Also obtain the equations of the lines of regression and obtain an estimate of y when x = 6.2. ( 0.95 , 13.14 )

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| X | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Y | 9 | 8 | 10 | 12 | 11 | 13 | 14 | 16 | 15 |

(2) The two regression lines are having their means and S.D. 31.6 , 38 and 3.72 ,6.31 and = - 0.36

 Find the two regression lines.

(3) In a partially destroyed laboratory record, only the lines of regression of y on x and x on y are available as 4x – 5y + 33 = 0 and 20x – 9y = 107 respectively. Calculate means of x and y; and coefficient of correlation. ( 13, 17, 0.6)

(3a) 7x – 16y +9 =0, 5y -4x – 3=0

(4) While calculating correlation coefficient between two variables x and y from 25 pairs of observations , the following results were obtained : . Later it was discovered at the time of checking that the pairs of values (8.12) and (6,8) were copied down as (6,14) and (8,6) respectively. Obtain the correct value of correlation coefficient. ( )

(5) Find the lines of regression and coefficient of correlation for the data given below:



(6) can 2x + 3y = 4 ; x – y = 5 be equations of valid regression lines?

(7) From 10 pairs of observations for x and y the following data is obtained : . Later it was found that two pairs of values ( 4,6 ) and ( 9,8 ) were copied down as ( 2,3 ) and ( 7,5 ). Obtain the correct value of correlation coefficient. (0.5532 )